COMPLEX NUMBERS

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THE UNIVERSITY OF MELBOURNE

Introduction of complex numbers

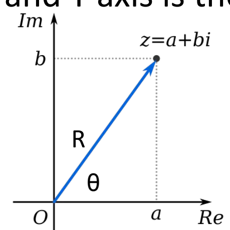
The discovery and usage of complex numbers was a crucial advancement for mathematics and physics. They are used as basis for a lot of modern fundamental theorems and formulas in **mathematics and physics.** Without complex numbers, certain calculations would be very **hard** to solve, and ultimately essential mechanisms such as measurement of AC current in **electrical engineering** or even Quantum Mechanics in **physics** wouldn't be easily usable or applicable.

Notations and visualisations of complex numbers

Complex numbers have both a **real component** and a **imaginary component.**

This is expressed in the polar form a+bi, where a is the **real component**, while b is the **imaginary component**.

This polar form can be graphed on a complex plane where X axis is the real number value, and Y axis is the imaginary number value.



By graphing this form, we can change the polar form of complex numbers into the exponential form

$$z = re^{i\theta}$$

History

Tartaglia:

Tartaglia was the second person to have solved the depressed cubic, other mathematicians were desperate to know how, especially Gerolamo Cardano. 25th of March 1539, he revealed the method after making Gerolamo Cardano swear to never reveal his method. Gerolamo Cardano started to work on the algorithm and after working on it he finally found the solution to the whole cubic equation. Without breaking his oath with Tartalia he found a way to publish his findings. He came across some cubic equations that are not so easy to solve because they contain square roots with negative numbers.

Gerolamo Cardano:

Complex numbers originally come from the cubic equation by Italians famous gambler, philosopher, physician and mathematician Gerolamo Cardano (1501 - 1576). Complex numbers were found inside the cubic equation, it was later called the imaginary number because of Rene Descartes who realised their utility.

References

How Imaginary Numbers Were Invented. (n.d.). Www.youtube.com. https://youtu.be/cUzklzVXJwo

tomrocksmaths. (2021, October 19). Complex Numbers, Air Traffic Control and RADAR. TOM ROCKS MATHS. https://tomrocksmaths.com/2021/10/19/air-traffic-control-and-radar/#:~:text=Furthermore%2C%20the%20radar%20is%20scanning

Complex numbers are essential in quantum theory, experiments reveal. (2022, January 11). Physics World. https://physicsworld.com

Practical implementations of complex numbers

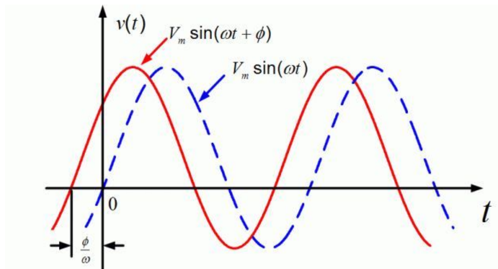
Mathematics

A lot of practical applications of imaginary numbers comes from the Euler's Rule, which is a mathematical way to convert imaginary numbers in exponential form into polar form(standard form) and vice versa. The exponential form is a simplified expression that can be used more easily in certain scenarios.

$$e^{ix} = \cos x + i \sin x$$

Electrical engineering

AC currents function by having a spinning magnet that pushes or attracts the electrons depending on its positions. This is why the flow of the electron alternates in a set interval due to the spinning magnet. The change of flow in the circuit can be graphed as a sin or cos graph, which all have a wave-like pattern.



Since the current can be presented using sin waves, the expression then becomes rsin(ωt+θ).The r and θ in this case are parallel to the coefficient of e and x respectively in the euler's formula. Similarly to what the euler's formula does, electrical engineers can use the same equation to convert values of a current(the sin graph) into values that are much easier to use (similar to the exponential form of complex numbers in maths) in certain situations.

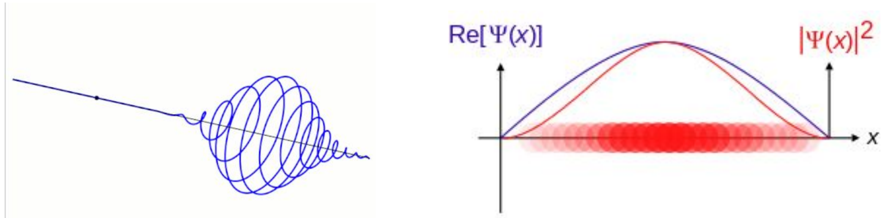
Quantum mechanics

Quantum mechanics is the study of how atomic or subatomic particles such as protons and electrons interact with energy.

The foundation of Quantum mechanics was based on the Schrödinger equation which contains the imaginary i as a constant value.

$$i\hbar \frac{\partial}{\partial t}|\psi(t)\rangle = \hat{H}|\psi(t)\rangle$$

The probability of finding a particle in a specific region is determined using wave functions.



Representations of wave functions

The Schrödinger equation is an physical representation of the probability a particle appears in a wave function, which makes it the very basis of modern quantum mechanics.