THE UNIVERSITY OF MELBOURNE

School of Mathematics and Statistics

2025 MATHEMATICS & STATISTICS RESEARCH CHALLENGE

Intermediate Booklet

- AU: Year 7 – 9
- NZ: Year 8 – 10

Overview

Students may select **one** research topic to investigate mathematically, in a team of up to 3 people or individually. Students should present their project and findings through one of the following:

1. Report (A4 or slideshow), or
2. Visual display (single-page poster or short video).

Students have until **6 August 2025** to conduct their research and submit a project. Projects must be in English.

The research topics in the booklet vary in their level of difficulty to allow all students to participate in the challenge. This is taken into account in the judging criteria which varies depending on the complexity of the chosen topic.

All research topics in this booklet have open-ended elements. **There is no requirement to provide complete responses to every prompt within a topic.** Students are encouraged to attempt as many prompts as they like, to the best of their ability. Projects that present mathematically interesting elements will be judged more favourably. The website provides an article with advice and guidance on how to conduct and supervise mathematical research[1].

Whenever explicit mathematics is involved, mathematical accuracy is vitally important, as is the correct use of mathematical language. In addition to these, the judging criteria include elements for originality, creativity, communication and presentation. Students are encouraged to go beyond the provided research prompts to also form their own questions and original investigations. If students use software or other people's work/findings in their project, the project must use proper referencing.

All teams who submit a valid research project will receive a certificate for participation, merit, high commendation, or distinction. The highest-quality projects in each age category will receive awards and cash prizes[2]. A team whose project is shortlisted for the Finals (report only) or for prizes (report and visual display) will be notified in October.

We hope everyone enjoys the challenge.

Good Luck!

[1] Read the article at: https://go.unimelb.edu.au/f8jp.
[2] Information about awards and prizes: https://go.unimelb.edu.au/23ns.

Topic I–01 Colouring Squares

Say we have a grid that needs to be filled with blue (solid) and pink (hatched) squares. We want to arrange the squares so that squares of one colour are always next to or surrounded by squares of a different colour (i.e. no same-coloured squares share an edge). For example, for a (1×4) grid, there are two such arrangements (Figure 1) and for a (2×2) grid, there are also two such arrangements (Figure 2).

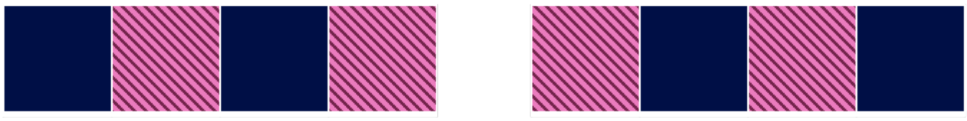

Figure 1: All possibilities using 2 colours on a (1×4) grid.

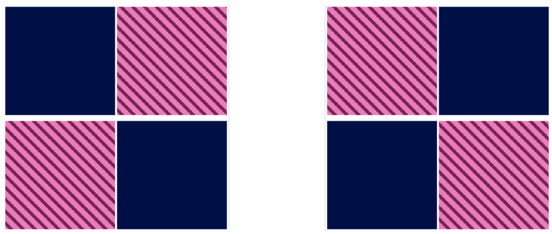

Figure 2: All possibilities using 2 colours on a (2×2) grid.

- How many ways are there to colour a (3×3) grid?
- With additional colours for the squares, how many different ways are there to colour (1×4), (2×2) and (3×3) grids?

You are welcome to explore this problem further by considering an $(m \times n)$ grid, where m and n are positive whole numbers.

- Investigate the number of ways to colour grids of different sizes. Comment on interesting observations.
- Is it possible to predict how many possibilities there are if you know the size of the grid and the number of colours available?

Topic I–02 Road Trip

Eddy's family is looking for a new car. They are weighing up various powertrain[1] options between electric, combustion, fuel cell, and hybrid. Eddy's family has two grown-ups, one teenager (Eddy), one toddler, and two medium-sized dogs. They live in Northland, a region in the North Island of Aotearoa New Zealand. In the future, they may move to Canterbury, a region in the South Island. Factors worth considering when buying a car include: price (buying outright vs financing), running costs (maintenance, electricity vs petrol), performance, and safety.

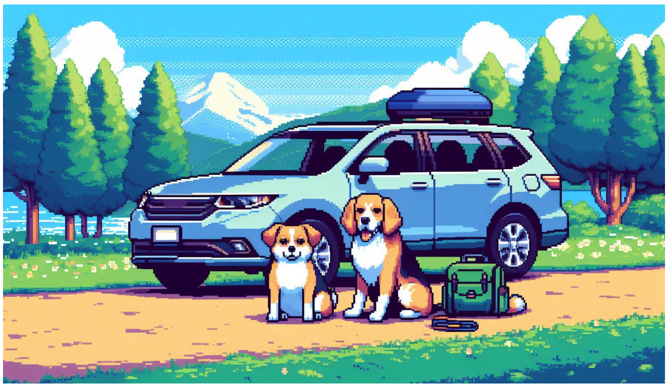

Image: Microsoft Designer.

Choose two cars with different powertrains (e.g. electric vs petrol) currently for sale as new[2]. Investigate one or more of the following areas for your chosen cars and use your findings to recommend a car for Eddy's family.

- When is it better to buy a car outright vs through a loan? Explain your assumptions about the family's financial situation and current interest rates.
- What type of powertrain has higher long-term cost? Make assumptions about the expected life of the chosen cars.
- What type of powertrain is suited to driving long distances (e.g. 600+ km)? Consider things like performance, public/private infrastructure.

You could extend this investigation by:

- Adding a car of a third powertrain type to the comparison.
- Comparing car travel with public transport.

[1] Powertrain in a car refers to the system that generates and delivers power to make the car move.
[2] You could base your investigation on cars owned by people you know and search for the cars' details online.

Topic I–03 Ferdy the Frog

Ferdy the Frog needs to cross a single lane road to get from her pond to the pond directly opposite to search for food (Figure 1). It's a one-way road only used by road trains, with 20 road trains per hour. The road trains are all 50 meters long and travel, on average, at 80 km/hour. It takes her 5 seconds to cross the road.

- Ferdy starts to cross at a random time. What is the probability she makes it across safely?

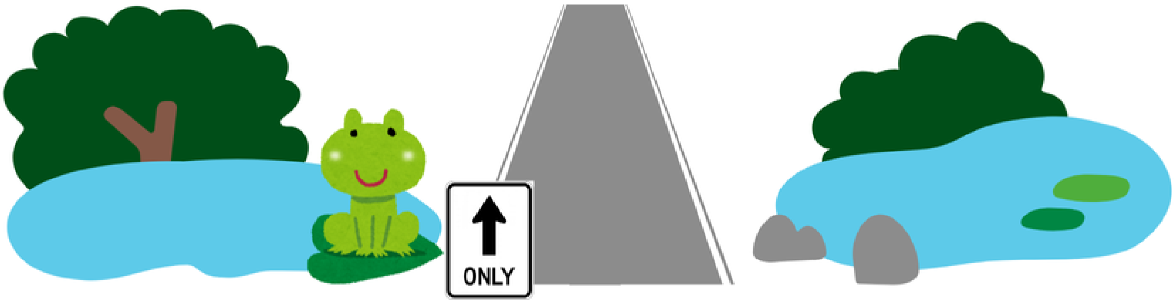

Figure 1: Ferdy the frog about to cross the road.

You could extend this investigation by considering some additional scenarios like:

- How would the probability change if the road had two lanes with road trains coming in both directions?
- If some parameters (e.g. frequency, length, more vehicle types, speed of the road trains) changed, what would change about the probability of Ferdy safely crossing the road?

Topic I–04 Stable Model

Consider the following four construction components:

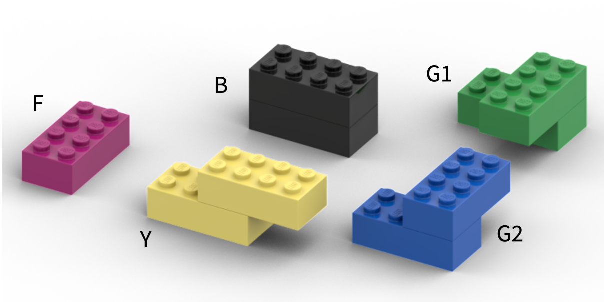

Figure 1: A base brick (F) and four components (B, Y, G1, G2).

Each component (B, Y, G1, G2) is made by joining two 4×2 bricks (F) on a 2×2 connection. Your task is to create[1] models using the available components and following these rules:

1. All connections in the model must be <u>**at least**</u> 2×2 (Figure 2).
2. Every component must be securely joined to at least one adjacent component (Figure 3).
3. The model must be stable (can stand on its own without external supports).
4. There are 20 components (5 each of B, Y, G1, G2); a model does not need to use all 20 components.

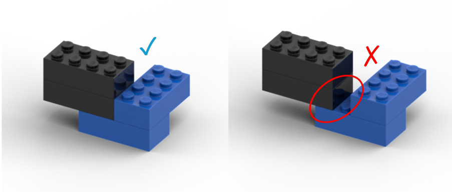

Figure 2: Left: legal connection 2×2. Right: illegal connection 1×2.

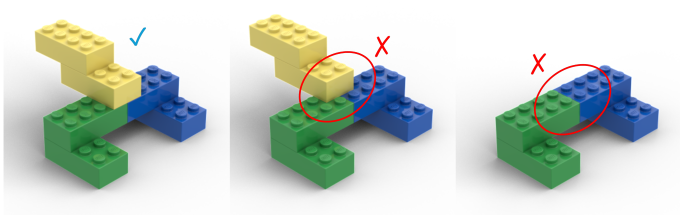

Figure 3: Left: components securely joined with legal connection 2×2. Middle: components joined with illegal connection 1×2. Right: components NOT securely joined.

[1] You may construct models using physical construction bricks or a free computer modelling software such as BrickLink Studio.

- Create models that have the largest possible:
    - Bird's eye view area
    - Surface area
    - Volume
- Describe how you measured the above.
- With 5 of each component (B, Y, G1, G2), is it possible to make a cuboid whose faces are smooth[2]?
    - If so, describe such a model.
    - If not, explain why or find out if it is possible with more copies of components.

You may like to extend this investigation along the following directions:

- Create models that satisfy your own set of rules.
- Come up with new components and revisit the above explorations.

[2] Smooth here means no protrusions or gaps along any face of the model.

Topic I–05 Bobbi's Book Club

Bobbi loves to read; they like books of all kinds. Each year Bobbi collects data on the top 50 selling books from an online bookstore as they are curious about any patterns in the book sales. Bobbi's data set contains the following information about each book:

- Title
- Author
- Rating (out of 5)
- Number of reviews
- Price (in dollars)
- Year
- Genre

Image: Microsoft Designer.

Using the data Bobbi collected[1], help them investigate the following:

- What book received the best rating from the most reviews in each year?
- Who is the most successful author at the bookstore? You should decide and explain what 'successful' means.
- Are there patterns in ratings, price, and genre over time? It may help to explore patterns visually.

You may like to extend your investigation in the following areas:

- Ask your own questions about Bobbi's data.
- Identify the sub-genre (e.g. fantasy, mystery, young adult, autobiography, self-help, science) for each book and revisit the above prompts.
- Expand your investigation to a larger data set[2].

[1] The data are available at: https://go.unimelb.edu.au/puz8.
[2] The larger data set is available at the above link.

Topic I–06 Ellipse

A circle is entirely defined by its radius, a, which has dimension 1. Therefore, without doing any calculations, its circumference must be Ca for some dimensionless number C, which does not depend on the radius. Similarly, the area of the circle must be related to a^2 because area is 2-dimensional. So the area is Da^2 for some dimensionless number D. Consider a stretched circle (an ellipse): it is determined by the two half-lengths, as shown in Figure 1.

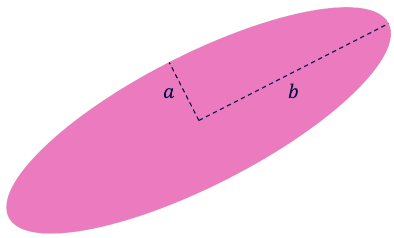

Figure 1: An ellipse with half lengths a and b.

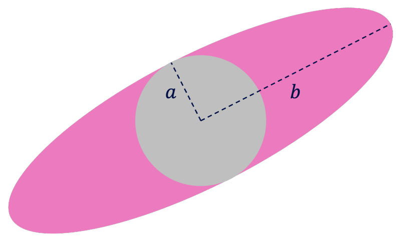

Figure 2: An ellipse with half lengths a and b, with an inscribed circle of radius a

Without knowing the formula for the area of an ellipse, we know the area must be larger than Da^2, since the circle fits entirely inside the ellipse (see Figure 2).

- Try to find an upper bound for the area of an ellipse. You should justify your answer.
- Try to determine a range for the perimeter of an ellipse. Explain your reasoning. Could you determine a more precise range?
- The volume of an ellipsoid is known. Can you provide some geometric arguments to arrive at the known formula?

You may explore the following extensions:

- Deducing or estimating the area of other shapes (e.g. isosceles or scalene triangles, kite, parallelogram) without doing any measurements.
- Why some shapes have simple formulas for properties like perimeter, area, or surface area, while others do not.

Topic I–07 Cycling Around

Consider the whole numbers from 0 up to $d-1$ arranged in increasing order, evenly spaced around a circle, starting with 0 at the top. Take a fixed whole multiplier m, where $0 < m \leq d$. For each n on the circle, draw a straight line with an arrow from n to $[n \times m]_d$. Here, $[x]_d$ is the remainder of x when divided by d.

For example, $16 \div 14 = 1\,r\,2$ and $4 \div 14 = 0\,r\,4$, so we say $[16]_{14} = 2$ and $[4]_{14} = 4$.

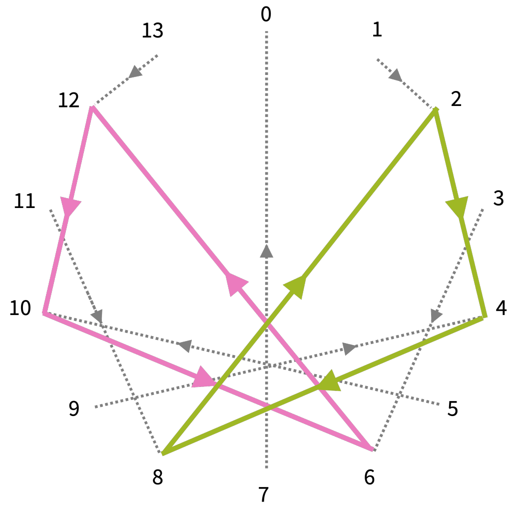

Figure 1: A diagram showing all the connections from n to $[n \times m]_d$ for $d = 14$ and $m = 2$. Cycles are coloured.

By following the above procedure, with $d = 14$ and $m = 2$, one can create a diagram as seen in Figure 1. Here are some notable features:

- Numbers like 2, 4, and 8 form a cycle of length three, while 6, 10 and 12 form a different cycle of length three.
- Some numbers like 1, 3 , 5, 9, 11, and 13 start outside of, but then become part of a cycle.
- Lastly, there is a path from 7 going to 0, which then stays there.

- Create a diagram for the values $d = 10$ and $m = 7$ and comment on any features you observe.
- Is there always guaranteed to be at least one cycle for any pair of values d, m? Provide an explanation.
- Can you devise a way to determine how many cycles there are, from a given d and m, without drawing all of the lines?
  - If you can determine there is a cycle, can you determine what length it will be?

Topic I–08 Tali's Tank

Tali is installing a system of pipes and tubes for their new fish tank. They purchased some equipment (Figure 1) from an online retailer. However, once the items arrived, Tali began to suspect that the geometry of the items may be flawed. Tali would like the rigid pipes, flexible tubes, and joints to be ideal in geometry, that is: the inner and outer diameters of pipes and tubes are perfectly circular along the entire length, and the joints are perfectly orthogonal with circular cross section.

Figure 1: Items Tali purchased online.

- Using only flat surfaces and a ruler, could Tali figure out whether the pipes and tubes have perfectly circular cross sections along their lengths? If so, describe how. If not, explain why.

Tali has some other tools including hose clamps, a vernier caliper, and tailor's tape measure (Figure 2) to help them evaluate the geometry of the equipment.

Figure 2: Tools Tali could use.

- Using the tools mentioned so far or any other sensible tool, describe a procedure that Tali could use to evaluate the geometry of the pipes, tube, and joints.
- You may like to extend this project by measuring different items with circular cross sections around your home, at school, or from the local hardware store and evaluating how circular they are.

Topic I–09 Coin Sequence

Consider the outcomes from a sequence of coin flips:

T H T T H T H T T T | H T T H T T T H H T

It is known that two different coins were used to generate this sequence; coin A has a probability of 0.3 to show a head and coin B has a probability of 0.7 to show a head. Suppose that the sequence was generated by flipping one of the coins 10 times, then the other coin 10 times.

- Determine which coin was used first and describe how you did it.

Now consider a different longer sequence of coin flip outcomes:

T T T H H T H H H H H T H H H H H H H T ...[1]

In this new scenario, you have information that

– Exactly two different coins were used.

– The coins were switched exactly once.

Given the above,

- Develop a method to estimate the probabilities of each coin showing heads and when the coins were switched. How sure are you about your findings?

To extend this problem further you may want to:

- Generate your own data, where you have knowledge of the parameters, and evaluate the accuracy of your method.
- Explore a more general case, where you don't have information about how many coins were used.

[1] The whole sequence can be downloaded at: https://go.unimelb.edu.au/puz8.

Topic I–10 Lucky Draw

You have won a lucky draw at a board game shop and you get to fill a bag with items. The bag can hold up to 11.5 kg; if the bag breaks, you don't get to keep any of the items. You can only select one of each item. The items you can choose from are:

Object	Weight (kg)	Value ($)
Lazuli	3.7	$56.40
Baeza	2.1	$44.20
Qatan	2.0	$52.50
Millennium: Silk Street	3.0	$61.70
Nicknames	2.9	$51.10
Inlets	3.9	$59.30
Outwitted	3.5	$54.60

- What items would you select to maximise the monetary value that you take, without exceeding the capacity of your bag? Explain why you chose those items.

To explore this prompt further, you may want to consider one or more of the following:

- How does the best collection of objects change when the capacity of the bag is changed?
- Additional constraints like packaging dimensions.
- Allowing for more than one of each item to be selected.